



COMSATS University Islamabad

Department of Computer Science

Terminal Examination – Spring 2022

Class: (BS) CS/SE/AI/DS/CYS

Course Code & Title: CSC102 Discrete Structures

Time Allowed: 3 Hrs

Instructors: Dr. Ahmed Kamran, Dr. Sheneela, Ms. Sajida Kalsoom

Semester: 1st / 2nd

Date: 30-06-2022

Max Marks: 50

Name: _____

Registration # 020

Instructions

1. Mobile phones are strictly prohibited in the examination hall.
2. Use black or blue ball-pen only. Markers/lead pencils are not allowed.

Question 1

CLO-1&2

Marks [7]

- 1) Use quantifiers and predicates with more than one variable to express these statements (3)
 - a) Some student in this class grew up in the same town as exactly one other student in this class.
 - b) Every student in this class has been in every building on campus.
 - c) There is a student in this class who owns a personal computer.
- 2) For each of these arguments, explain which rules of inference are used for each step. (4)
 - a) "Everyone in New Jersey lives within 50 miles of the ocean. Someone in New Jersey has never seen the ocean. Therefore, someone who lives within 50 miles of the ocean has never seen the ocean."
 - b) "All movies produced by John Sayles are wonderful. John Sayles produced a movie about coal miners. Therefore, there is a wonderful movie about coal miners."

Question 2

CLO-3

Marks [14]

1. Suppose that the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Find the set specified by each of these bit strings. (1)

01 0111 1000
2. Given two functions: $f = \{(-2, 1), (0, 3), (4, 5)\}$ and $g = \{(1, 1), (3, 3), (7, 9)\}$, find $(g \circ f)$ and determine its domain and range. (3)

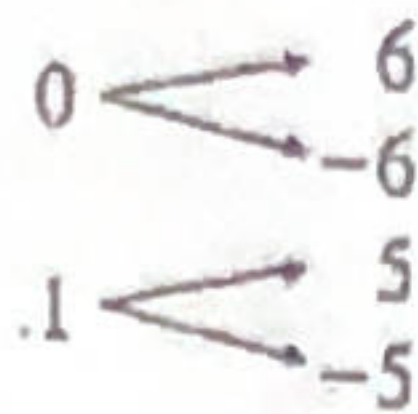
x is a movie

1

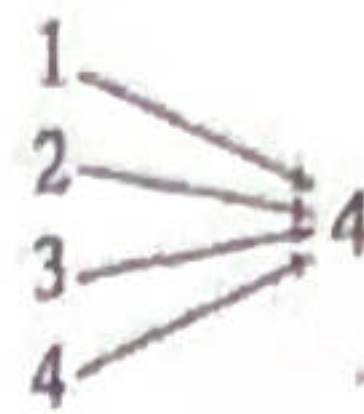
$P \rightarrow Q$
 $P \rightarrow R$

3. Decide whether the relation is a function. If it is a function, give the domain and range.
(1)

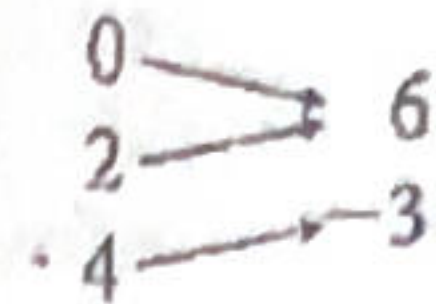
1. Input Output



2. Input Output



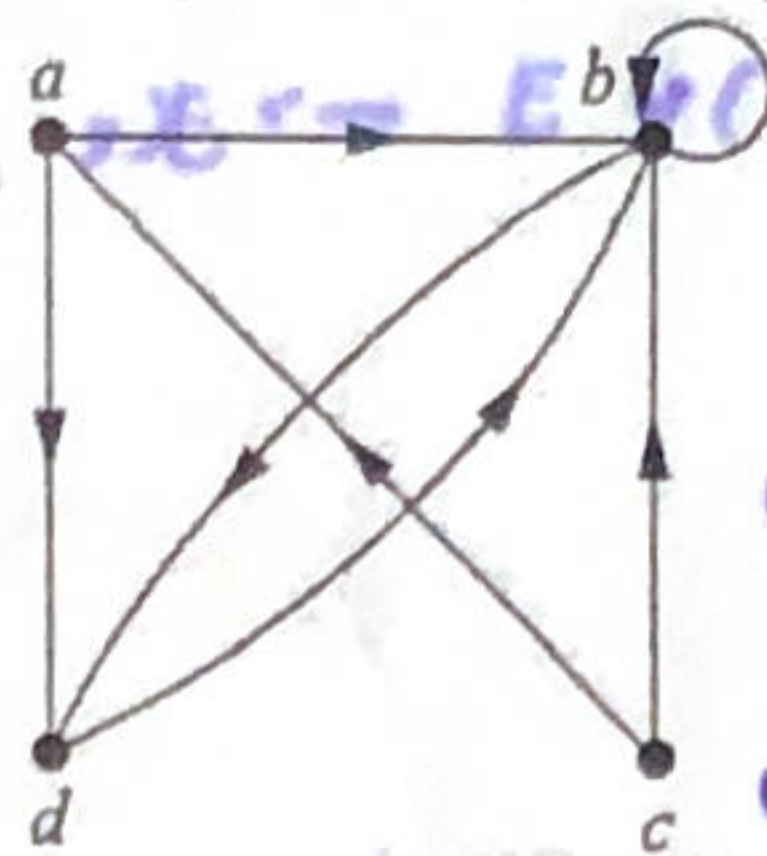
3. Input Output



4. What is the closed-form definition for the sequence 1, -3, 9, -27, ...? (2)
5. Let $R_1 = \{(1, 2), (2, 3), (3, 4)\}$ and $R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$ be relations from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$. Find (3)

- $R_2 - R_1$
- $R_2 \circ R_1$
- $(R_1 - R_2)^{-1}$

6. Determine whether the relations for the directed graphs shown in Figure are reflexive, symmetric, antisymmetric, and/or transitive; (2)



not reflexive
not symmetric
Anti symmetric

$(a,b) \rightarrow (a,b)$
 $(a,d) \rightarrow (a,b)$
 $(b,b) \rightarrow (b,b)$
 $b,b \rightarrow (b,d)$
 $c,b \rightarrow c,b$
 $c,d \rightarrow c,b$
 $c,d \rightarrow c,b$
 $d,b \rightarrow d,b$
 $d,d \rightarrow d,d$

$(a,d), (a,b), (b,b), (b,d), (c,d), (c,b), (d,b)$
 (a,a) a
 $(a,b), (a,d), (b,b), (b,d), (c,a), (c,b), (d,b)$
 (a,b) a

7. A relation R is defined on the set of positive integer as xRy if $2x+y \leq 5$ is reflexive, transitive, symmetric? (1)
8. Compute the double summation. (1)

$$\sum_{i=0}^2 \sum_{j=1}^3 ij$$

18

Question 3

CLO-4

Marks [7]

- 1) The harmonic numbers $H_j, j = 1, 2, 3, \dots$, are defined by $H_j = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{j}$

Use mathematical induction to show that

$$H_{2^n} \geq 1 + \frac{n}{2}, \text{ whenever } n \text{ is a nonnegative integer.}$$

(4)

2) Prove that if x is irrational, then $\frac{1}{x}$ is irrational.

$$\frac{1}{x} \text{ is } \frac{x}{1} \cdot \frac{1}{x} \quad (3)$$

Question 5

CLO-5

Marks [12]

✓ A combination lock requires three selections of non repeating numbers, first from 1 through 30, second from 30 through 70 and third from 1 through 20. (3)

a. How many different combinations are possible?

b. Suppose the locks are constructed in such a way that the repetition in numbers is allowed. How many different combinations are possible?

2) What is the coefficient of x^9 in $(2-x)^{19}$? (1)

3) Use the Binomial Theorem to expand $(4x^4 + 3y^2)^4$. (2)

4) How many ways are there to select 11 players from a 20 member hockey team to make a trip to a match at another university. (2)

5) What is the probability that a positive integer not exceeding 100 selected at random is divisible by 5 or 7? (2)

6) What is the probability that when a coin is flipped six times in a row, it lands heads up every time? (2)

1 2 3 4 5 6

h t

h t

h t

Question 5

CLO-6

Marks [10]

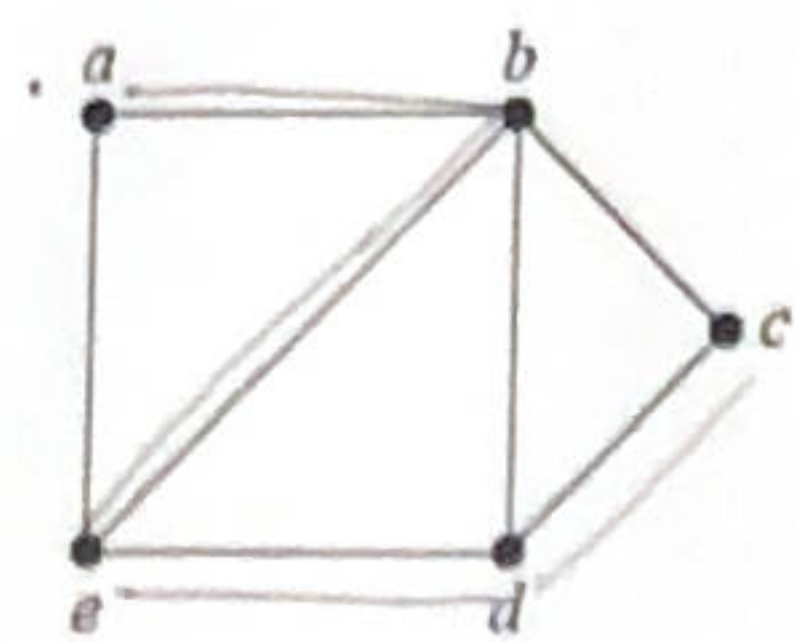
1) a) Evaluate the following prefix expression.
+ = ↑ 3 2 ↑ 2 3 / 6 - 4 2

b) Evaluate the following given postfix expression.
5 2 1 - 3 1 4 ++ *

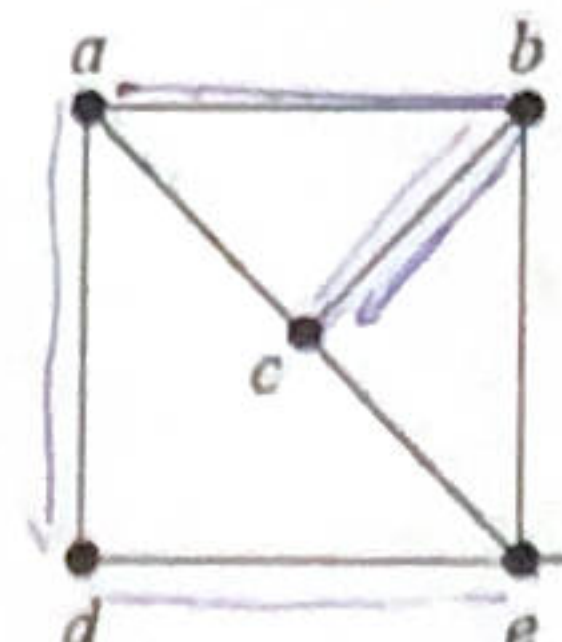
2) Construct the ordered rooted tree whose preorder traversal is $a, b, f, c, g, h, i, d, e, j, k, l$, where a has four children, c has three children, j has two children, b and e have one child each, and all other vertices are leaves. (2)

3) How many edges are there in an undirected graph with 10 vertices each of degree six? (2)

4) Determine whether the given graphs have Hamilton circuit. If it does, find such a circuit. If no Hamilton circuit exists, determine whether the graph has an Hamilton path and construct such a path if one exists. (2)

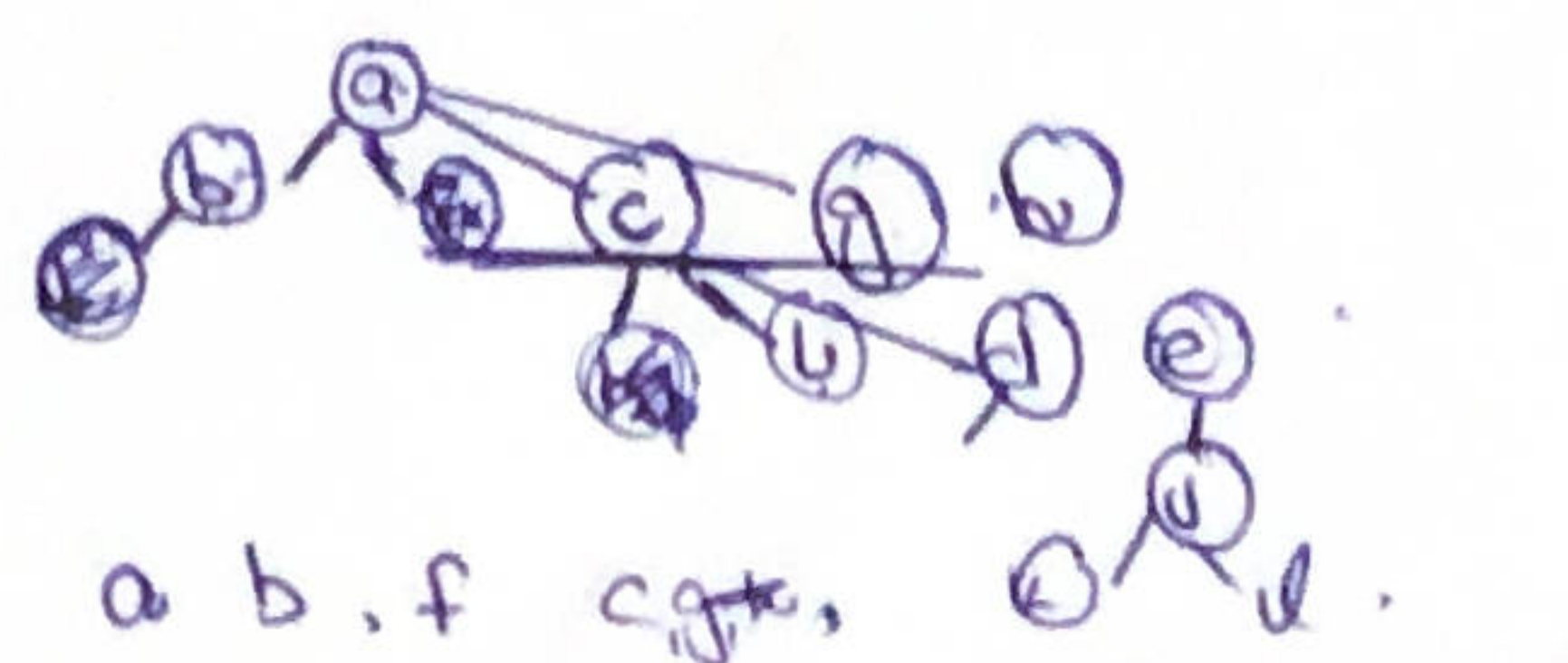


G1



G2

3

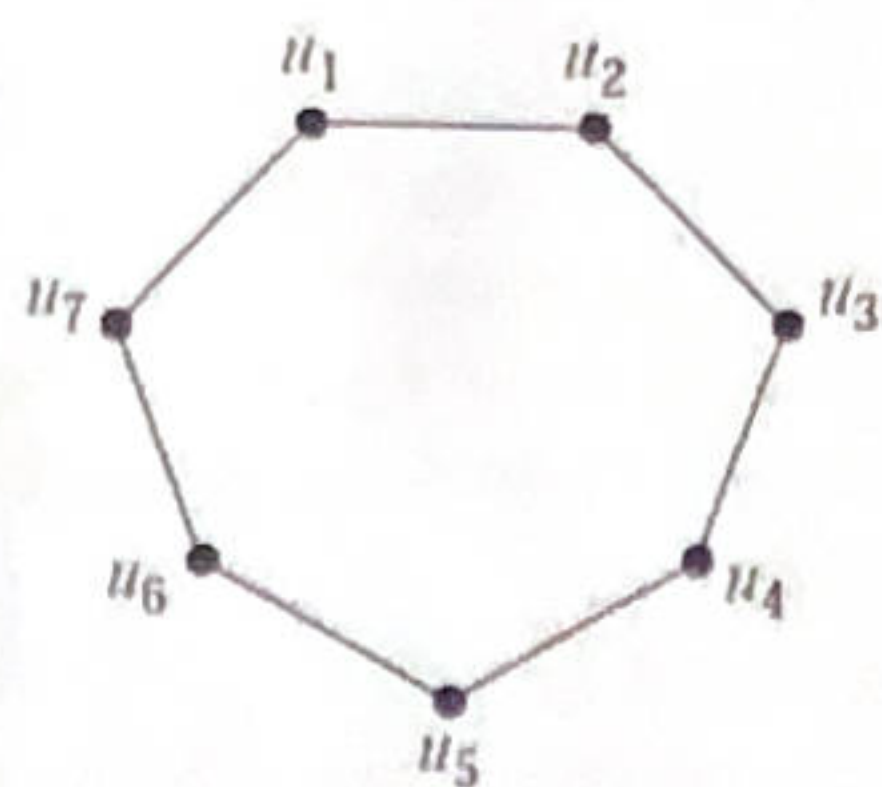
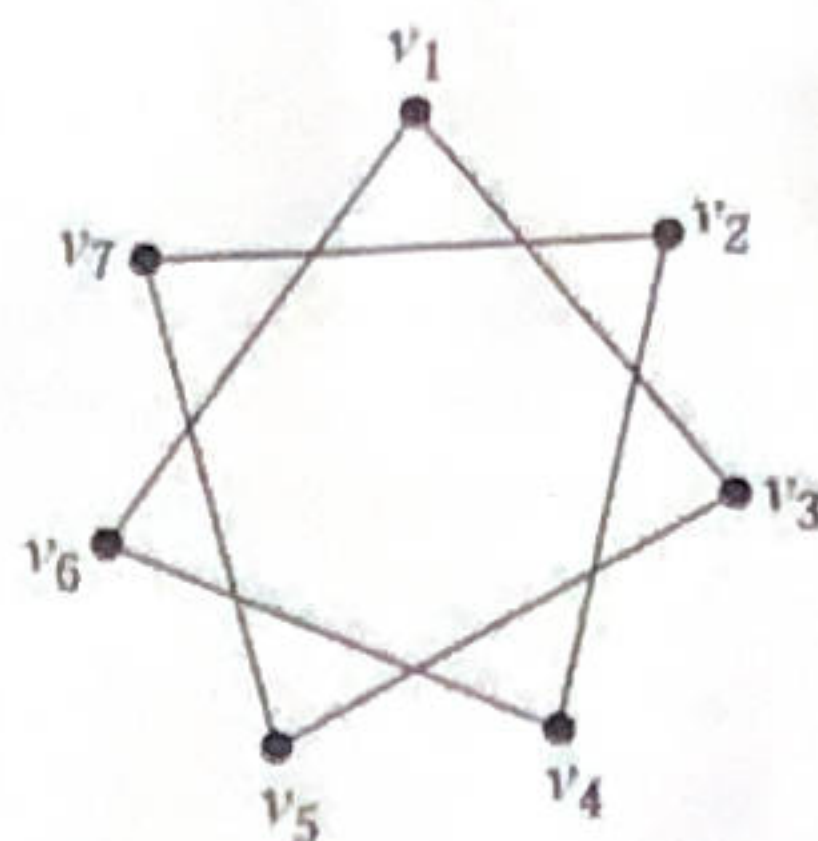


a b, f c g, h i j k l

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$

6

- 5) Are the following graphs, U & V, isomorphic? Provide an explanation about their vertices, edges, vertices connectivity and degree of vertices. (2)



Good Luck ☺

Cheat Sheet

TABLE 6 Logical Equivalences.	
Equivalence	Name
$p \wedge T \equiv p$ $p \vee F \equiv p$	Identity laws
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$	Negation laws

TABLE 1 Rules of Inference.		
Rule of Inference	Tautology	Name
$\frac{p \quad p \rightarrow q}{\therefore q}$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus ponens
$\frac{\neg q \quad p \rightarrow q}{\therefore \neg p}$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus tollens
$\frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$\frac{p \vee q \quad \neg p}{\therefore q}$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive syllogism
$\frac{p}{\therefore p \vee q}$	$p \rightarrow (p \vee q)$	Addition
$\frac{p \wedge q}{\therefore p}$	$(p \wedge q) \rightarrow p$	Simplification
$\frac{p \quad q}{\therefore p \wedge q}$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$\frac{p \vee q \quad \neg p \vee r}{\therefore q \vee r}$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution

TABLE 2 Rules of Inference for Quantified Statements.	
Rule of Inference	Name
$\frac{\forall x P(x)}{\therefore P(c)}$	Universal instantiation
$\frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$	Universal generalization
$\frac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$	Existential instantiation
$\frac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$	Existential generalization

TABLE 7 Logical Equivalences Involving Conditional Statements.	
$p \rightarrow q \equiv \neg p \vee q$	
$p \rightarrow q \equiv \neg q \rightarrow \neg p$	
$p \vee q \equiv \neg p \rightarrow q$	
$p \wedge q \equiv \neg(p \rightarrow \neg q)$	
$\neg(p \rightarrow q) \equiv p \wedge \neg q$	
$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$	
$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$	
$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$	
$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$	

TABLE 8 Logical Equivalences Involving Biconditional Statements.	
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$	
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$	
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$	