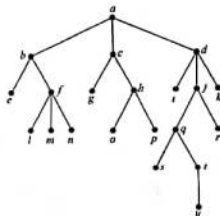
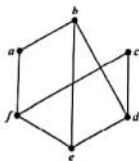


1) Answer these questions about the rooted tree illustrated.

- Which vertex is the root?
- Which vertices are internal?
- Which vertices are leaves?
- Which vertices are children of j ?
- Which vertex is the parent of h ?
- Which vertices are siblings of o ?
- Which vertices are ancestors of m ?
- Which vertices are descendants of b ?



2) Apply graph coloring and determine whether the following graph is a bipartite or not.



- 3) Determine whether the given graphs have an Euler circuit. Construct such a circuit when exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

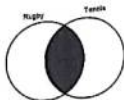
Question 1**CLO-1 & 2****Marks [3+4]**

- 1) Express each of these statements using predicates, quantifiers and logical connectives.
 - a) A man qualifies for the marathon if his best previous time is less than 3 hours and a woman qualifies for the marathon if her best previous time is less than 3.5 hours.
 - b) There is a student who has taken more than 21 credit hours in a semester and received all A's.
 - c) The system mailbox can be accessed by everyone in the group if the file system is locked.
- 2) For each of these arguments, explain which rules of inference are used for each step.
 - a) "Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job."
 - b) "Linda, a student in this class, owns a red convertible. Everyone who owns a red convertible has gotten at least one speeding ticket. Therefore, someone in this class has gotten a speeding ticket."

Question 2**CLO-****Marks [2+1+2+2+1+2+2+2]**

1. Let f be the function from $\{a, b, c\}$ to $\{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$, and $f(c) = 1$. Is f invertible, and if it is, what is its inverse?
2. Is the function $f(x) = x^2$ from the set of integers to the set of integers onto?

3. In a class of 80 students, there are 60 students who play rugby and 35 who play tennis. What number would go in the coloured section of this diagram, if it is known all students play either rugby, tennis?



4. If there were only 3 types of music in the world: pop, rock and rap, and set A equals all the people in the world who prefer pop music, what is A^c ?
5. What is the Cardinality of the Power set of the set $\{0, 1, 2\}$.
6. Find the first five terms of the sequence defined by each of these recurrence relations and initial conditions.

$$a_n = a_{n-1} + a_{n-2}, \quad a_0 = 1, \quad a_1 = 2, \quad a_2 = 0$$

7. Find the sum of the geometric series $2 + 6 + 18 + 54 + \dots$, where there are 6 terms in the series.
8. Consider the relation R on a set $\{1, 2, 3, 4, 5\}$.
 $R = \{(1,1), (1,3), (1,5), (2,2), (2,4), (3,1), (3,3), (3,5), (4,2), (4,4), (5,1), (5,3), (5,5)\}$.
 Explain whether the relation 'R' is an equivalence relation or not?

Question 3**CLO-4****Marks 13+41**

- 1) Use mathematical induction to prove that

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2 (n+1)^2}{4} \quad \text{for all positive integers } n.$$

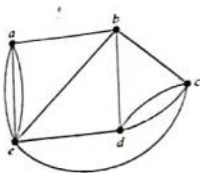
- 2) Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

Question 5**CLO-5****Marks 13+2+2+2+31**

1. In a running race of 12 sprinters, each of the top 5 finishers receives a different medal. How many ways are there to award the 5 medals?
2. How many license plates can be made using either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits?
3. Find the tenth term in the expansion of $(x + y)^{13}$?
4. What is the probability that a randomly selected integer chosen from the first 100 positive integers is odd?
5. An airline reports that 94% of its flights arrive on time. What is the probability that the flight is late.



G1



G2

- 4) Are the following graphs, U & V, isomorphic? Provide an explanation about the edges, vertices connectivity and degree of vertices.

