

SP21-BCS-028

Hemant Kumar



COMSATS University Islamabad
Islamabad Campus Fall 2021

Course: MTH104 - Calculus & Analytic Geometry
Level: BCS-II, BCT-II, BSE-II

Instructors: Dr. Muhammad Yousaf
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January 06, 2022
Total Marks: 50
Time Allowed: 3 Hours

Instructions:

Answer ALL questions, show all the work, answers without explanations will get partial credits only.

Q. No. 01: (04 marks) Show that

$$f(x) = \begin{cases} x^2, & x \leq 1 \\ \sqrt{x}, & x > 1 \end{cases}$$

is continuous at $x = 1$. Determine whether f is differentiable at $x = 1$. If so, find the value of the derivative there.

Q. No. 02: (05 marks) Use implicit differentiation to find the slope of the tangent line to the curve at the specified point

$$y^3 + yx^2 + x^2 - 3y^2 = 0; \quad (0, 3)$$

Q. No. 03: (5 marks) Find the limit

$$\lim_{x \rightarrow +\infty} \frac{e^{3x}}{x^3}$$

Q. No. 04: (5 marks) Sketch the graph. Find the intervals on which f is increasing, the intervals on which f is decreasing, the open intervals on which f is concave up, the open intervals on which f is concave down, and the x-coordinates of all inflection points.

$$f(x) = x^2 - 3x + 8.$$

Q. No. 05: (5 marks) A rectangular plot of land is to be fenced in using two kinds of fencing. Two opposite sides will use heavy-duty fencing selling for \$3 a foot, while the remaining two sides will use standard fencing selling for \$2 a foot. What are the dimensions of the rectangular plot of greatest area that can be fenced in at a cost of \$600?

Q. No. 06: (5 marks) Evaluate

$$\int \frac{\sin \theta}{\cos^2 \theta + 1} d\theta.$$

Q. No. 07: (5 marks) Sketch the region enclosed by the curves and find its area

$$y = x^2, y = x^3.$$

Q. No. 08: (5 marks) Find the volume of the solid generated when the region between the graphs of the equations $f(x) = \frac{1}{2} + x^2$ and $g(x) = x$ over the interval $[0, 2]$ is revolved about the x -axis.

Q. No. 09: (5 marks) Check the convergence and monotonicity of the sequence

$$\{\tan^{-1} n\}_{n=1}^{\infty}.$$

Q. No. 10: (5 marks) Use any method to determine whether the series converges

$$\sum_{k=1}^{\infty} \frac{k!10^k}{3^k}.$$

Good Luck