

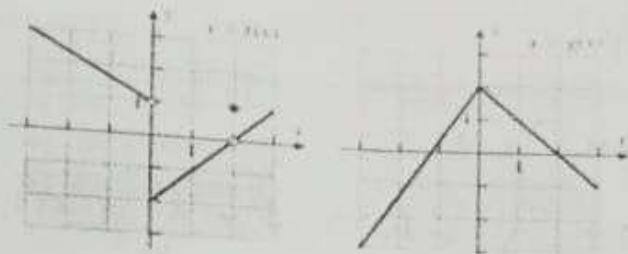


COMSATS University Islamabad (CUI)
Mid Term Exam (Fall 2021)

Program: BCS (IIA)
Subject: Calculus and Analytical Geometry
Instructor: Ms. Itrat Rubab

Marks: 25
Time Allowed: 1hr & 30mins
Date: November 18th, 2021.

Q No.1:



From the above graph, find

- a) $\lim_{x \rightarrow 0^+} [f(x) + g(x)]$
b) $\lim_{x \rightarrow 0^-} [f(x) + g(x)]$

Q No.2: Find the limit. $\lim_{x \rightarrow +\infty} (\sqrt{x^2 - 3x} - x)$

Q No.3: Find the limit, $\lim_{\theta \rightarrow 0^+} [\ln(\sin 2\theta) - \ln(\tan \theta)]$

Q No.4: Find the inverse of the given function also find the domain and range of f and f^{-1}
 $f(x) = (e^x)^2 + 1$

Verify that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$

Q No.5: Find a nonzero value for the constant k that makes

$$f(x) = \begin{cases} \frac{\tan kx}{x}, & x < 0 \\ 3x + 2k^2, & x \geq 0 \end{cases}$$

continuous at $x=0$

Good Luck

Handwritten calculations for Q No.2:

$$\begin{aligned} & \lim_{x \rightarrow +\infty} (\sqrt{x^2 - 3x} - x) \\ &= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 - 3x} - x)(\sqrt{x^2 - 3x} + x)}{\sqrt{x^2 - 3x} + x} \\ &= \lim_{x \rightarrow +\infty} \frac{-3x}{\sqrt{x^2 - 3x} + x} \\ &= \lim_{x \rightarrow +\infty} \frac{-3}{\sqrt{1 - \frac{3}{x}} + 1} \\ &= \frac{-3}{1 + 1} = -1.5 \end{aligned}$$

Handwritten calculations for Q No.4:

$$\begin{aligned} & f(x) = (e^x)^2 + 1 \\ & y = (e^x)^2 + 1 \\ & y - 1 = (e^x)^2 \\ & \sqrt{y - 1} = e^x \\ & \ln(\sqrt{y - 1}) = x \\ & f^{-1}(y) = \ln(\sqrt{y - 1}) \end{aligned}$$

Handwritten calculations for Q No.5:

$$\begin{aligned} & \text{For } x < 0, \quad \frac{\tan kx}{x} \rightarrow \frac{k}{1} = k \\ & \text{For } x \geq 0, \quad 3x + 2k^2 \rightarrow 2k^2 \\ & \text{For continuity at } x=0, \quad k = 2k^2 \\ & \Rightarrow k = 0 \text{ or } k = \frac{1}{2} \end{aligned}$$

Handwritten calculations for Q No.5:

$$\begin{aligned} & 1 + 2(1) \left(\frac{1}{2} \right) = 1 + 1 = 2 \\ & + \frac{2}{1} + \frac{1}{1^2} + 1 = 2 + 2 + 1 = 5 \end{aligned}$$