COMSATS University Islamabad
Islamabad Campus,

Terminal Examination FA-2022

Program: BEE-1(A,B,E), EEE-1A
Subject: Calculus & Analytical Geometry (MTH-104)
Instructor: Dr. Shagufta, Dr. Saif ur Rehamn

Max Marks: 50
Time Allowed: 3 Hours.
Date: 26th Jan 2023

All questions carry equal marks.

Q.1. [CLO-6, PLO2] Find the volume of the solid that results when the region enclosed by the curves $y = \cos x$, $y = \sin x$, $x = 0, x = \frac{\pi}{4}$, is revolved about x-axis.

Q.2. [CLO-6, PLO2] Sketch the region enclosed by the curves
$y = x, y = 4x, y = -x + 2$ and find its area.

Q.3. [CLO-6, PLO2] Evaluate the integral using the Fundamental Theorem of Calculus (if necessary) $\int_1^2 \frac{dx}{x^2-6x+9}$.

Q.4. [CLO-6, PLO2] Find the exact arc length of the curve $y = \frac{(x^6+8)}{16x^2}$ from $x = 2$ to $x = 3$.

Q.5. [CLO-5, PLO2] Find the intervals on which $f(x) = \frac{x^2}{x^2+2}$ is increasing, decreasing, concave up, concave down, and the x-coordinate of all inflection points.

Q.6. [CLO-5, PLO2] Determine whether all hypothesis of Rolle's Theorem are satisfied for $f(x) = \sqrt{4 - x^2}$ on the interval $[-2, 2]$. If so, find all values of c guaranteed in the conclusion of the theorem. If not, state which hypothesis fails.

Q.7. [CLO-4, PLO1] A rectangular field is to be bounded by a fence on three sides and by a straight stream on the fourth side. Find the dimensions of the field with maximum area that can be enclosed using 1000 ft of fence.

Q.8. [CLO-4, PLO1] a) Explain the difference between average and instantaneous rate of change and discuss how they are calculated.
b) The amount of water in a tank t minutes after it has started to drain is given by

$$W = 100(t - 5)^2 \text{ gal}$$

i) At what rate is the water running out at the end of 5 min?
ii) What is the average rate at which the water flows out during the first 5 min?

Q.9. [CLO-3, PLO1] Sketch the graph of the function and check its continuity at $x = -5$ and $x = 5$

$$f(x) = \begin{cases} -1, & x \le -5 \\ \sqrt{25 - x^2} & -5 < x < 5 \\ x - 5 & x \ge 5 \end{cases}$$

Q.10. [CLO-2, PLO1] Find the limits i) $\lim_{x\to 0} \frac{x^2 - 3\sin x}{x}$ ii) $\lim_{t\to \pi/4} \frac{\cos t - \sin t}{t - \pi/4}$