

Instructions: -

- i. Answer all questions..
- ii. All questions carry equal marks.
- iii. Make sure the question numbers in your answer book align with those on the question paper.

Q.1: Use differentiation to show that the given sequence is strictly increasing or strictly decreasing.

$$\left\{ \frac{\ln(n+2)}{n+2} \right\}_{n=1}^{n=+\infty}$$

Q.2: Evaluate the integrals.

(i) $\int \frac{x}{\sqrt{1-x^4}} dx$ (ii) $\int_{-1}^1 \ln(x+2) dx$

Q.3: Sketch the region enclosed by the curves and find its area.

$$x^2 = y, \quad x = y - 2$$

Q.4:

- (i) Express the sum of the odd integers from 1 to 100 in sigma notation.
- (ii) Express the sum in closed form.

$$\sum_{k=1}^n \left(\frac{5}{n} - \frac{2k}{n} \right)$$

Q.5: A rectangle has its two lower corners on the x-axis and its two upper corners on the curve $y = 16 - x^2$. For all such rectangles, what are the dimensions of the one with largest area?

Q.6: Use any method to find the relative extrema of the function f .

$$f(x) = x^4 - 12x^3$$

Q.7: A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate of $\frac{3ft}{s}$. How rapidly is the area enclosed by the ripple increasing at the end of 10 s? (Area, $A = \pi r^2$)

Q. 8: Find $\frac{d^2y}{dx^2}$ by implicit differentiation.
 $x \cos y = y$

Q. 9: Find an equation for the tangent line to the graph at the specified value.
 $y = \sec^3(\frac{\pi}{2} - x), \quad x = -\frac{\pi}{2}.$

Q. 10: Let

$$g(t) = \begin{cases} t - 2, & t < 0 \\ t^2, & 0 \leq t \leq 2 \\ 2t, & t > 2 \end{cases}$$

Find

(a) $\lim_{t \rightarrow 0} g(t)$

(b) $\lim_{t \rightarrow 1} g(t)$

(c) $\lim_{t \rightarrow 2} g(t)$