



Terminal – SP 2014

Course Title:	Calculus and Analytic Geometry	Course Code:	MTH104	Credit Hours:	3(3,0)
Course Instructor/s:	Dr. Ayesha Sohail, Dr. Hani Shaker, Dr. Imran Ahmed, Dr. Muhammad Hussain, Dr. Sarfraz, Atiq-ur-Rehman, Maqsood Ahmad.	Program Name:	BCS, BSE, BTE, BEE, BCE		
Semester:	1 st	Batch:	SP-14	Section:	All
Time Allowed:	03:00 Hours (Objective 30 min+ Subjective 2:30 Hours)		Maximum Marks:	100 (Objective 20+ Subjective 80)	
Student's Name:			Reg. No.		

Important Instructions / Guidelines:

- Attempt all the questions on the separate answer booklet provided.
- Answers that have been altered after cutting, eradicating or over writing in Objective Section will not be considered.

Objective Part

MCQ: 1. The domain of the function $f(x) = \sqrt{x-1}$ is (1)

- (a) $\mathbb{R}^+$. (b) $[1, \infty)$. (c) $[-1, \infty)$. (d) $\mathbb{R}^-$.

MCQ: 2. The range of $f(x) = \frac{1}{x^2}$ is (1)

- (a) $\mathbb{R}$ (b) 1 (c) $[0, \infty)$ (d) $(0, \infty)$

MCQ: 3. Consider the functions $f(x) = x^2$, $g(x) = x + 1$. What is the value of $f \circ (g \circ f)(2)$? (1)

- (a) 24. (b) 25. (c) 26. (d) None of these.

MCQ: 4. The value of the sum $\sum_{k=1}^{10} 10$ is one of the following. (1)

- (a) 10. (b) 20. (c) 100. (d) None of these.

MCQ: 5. Which one of the following function is an odd function? (1)

- (a) $f(x) = \cos x$. (b) $g(x) = x^2 + \sin x$
(c) $h(x) = \tan x$ (d) $q(x) = 2$.

MCQ: 6. Which one of the following statement is not true? (1)

- (a) The domain of $f(x) = \cos x$ is $\mathbb{R}$.
(b) $|x|$ is differentiable everywhere.
(c) An even function is symmetric about y -axis.
(b) The range of $f(x) = 3$ is 3.

MCQ: 7. 8th term of the sequence $\left\{ \frac{(n-3)^2}{5} \right\}$ is (1)

- (a) 5. (b) 25. (c) 8. (d) 0.

MCQ: 8. $f(x) = \frac{x^2}{x-2}$ is discontinuous (1)

- (a) only at $x = 2$. (b) at $x = 0$ and at $x = 2$.
(c) Everywhere. (d) Nowhere.

MCQ: 9. The series (1)

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

- (a) converges if $p \geq 1$. (b) diverges if $p \geq 1$.
(c) converges if $p > 1$. (d) converges if $p < 1$.

MCQ: 10. Third order derivative of $f(x) = e^{2x}$ is (1)

- (a) e^{2x} (b) $2e^{2x}$ (c) $4e^{2x}$ (d) $8e^{2x}$

MCQ: 11. The value of the integral $\int_0^1 |x| dx$ is (2)

- (a) 0 (b) $\frac{1}{2}$. (c) $\frac{-1}{2}$. (d) 2.

MCQ: 12. The value of the sum $\sum_{k=1}^{\infty} \frac{2}{3^{k-1}}$ is (2)

- (a) 3 (b) $\frac{2}{3}$. (c) $\frac{1}{3}$. (d) 0.

MCQ: 13. Area of the region bounded between one arch of $f(x) = \sin x$ and x -axis is (2)

- (a) 2π . (b) $\frac{1}{2}$. (c) 2. (d) 0.

MCQ: 14. (2)

$$\lim_{x \rightarrow \infty} \left\{ \frac{x^2 - x + 2}{x^2 - x} \right\}$$

- (a) Does not exist. (b) Exists and is equal to 1.
(c) Exists and is equal to 2. (d) Exists and is equal to -2.

MCQ: 15. (2)

$$\lim_{x \rightarrow 0} \frac{\tan 2x}{x}$$

- (a) 1. (b) 2 (c) $\frac{1}{2}$. (d) Does not exist.

(Objective Solutions)

Question	Answer
1	<i>b</i>
2	<i>d</i>
3	<i>b</i>
4	<i>c</i>
5	<i>c</i>
6	<i>b</i>
7	<i>a</i>
8	<i>a</i>
9	<i>c</i>
10	<i>d</i>
11	<i>b</i>
12	<i>a</i>
13	<i>c</i>
14	<i>b</i>
15	<i>b</i>



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<ul style="list-style-type: none">Attempt all questions. Justify your answer with proper reasoning.								

Question: 1. **(10+8+7)**

- a) Determine the values of constants "A" and "B" so that following function is continuous at every real number and then sketch its graph.

$$f(x) = \begin{cases} 3x & \text{if } x \leq 2 \\ Ax + B & \text{if } 2 < x < 5 \\ -6x & \text{if } x \geq 5 \end{cases}$$

- b) Find the equations of the tangent and normal lines to the graph of the implicit function determined by the equation $\pi \tan(x - y) = 2x$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$.

- c) Find $\frac{d^n y}{dx^n}$ of the function $y = \frac{1}{5}x^5 + x^4 + 3x^3 + 10x^2 + 100 + e^{ax}$ for some constant "a".

Question: 2. **(10+8+7)**

- a) Sketch the graphs of $f(x) = x^2 + 1$ and $g(x) = 4x + 1$. Find the point of intersections of the two graphs. Find the area of the region bounded by $f(x)$ and $g(x)$

- b) Evaluate the integrals: (i). $\int \frac{x}{x^4 + 1} dx$, (ii). $\int_{-3}^{-1} \frac{x^2}{\sqrt{1-x}} dx$

Question: 3. **(10+10)**

- a) Find the center, radius and the interval of convergence of the power series

$$\sum_{k=1}^{\infty} \frac{1.5.9 \dots (4k-3)}{2.4.6 \dots (2k)} (x+6)^k.$$

- b) By appropriate method check the convergence and divergence of the series

$$\sum_{k=1}^{\infty} (-1)^{k+1} \left(\frac{k+2}{3k+1} \right)^k.$$

If it converges, determine whether the convergence is absolute or conditional.

Question: 4. **(5+5)**

Evaluate: (i). $\int_0^{\infty} x^2 e^{-x^3} dx$ and (ii). $\lim_{x \rightarrow \infty} \frac{1 - \cos(2/x)}{\tan(3/x)}$

Solution Key

Question: 1.

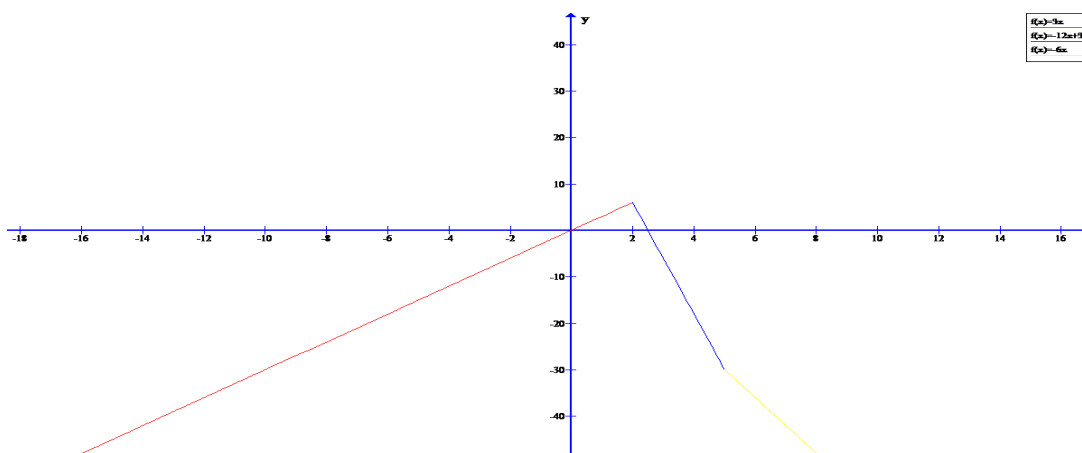
a)

As f is continuous at $x = 2$ and $x = 5$. Therefore, $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2)$

$$\Rightarrow 2A + B = 6 \quad (1)$$

$$\lim_{x \rightarrow 5^-} f(x) = \lim_{x \rightarrow 5^+} f(x) = f(5) \Rightarrow 5A + B = -30 \quad (2)$$

from (1) and (2), we get: $A = -12$ and $B = 30$.



b)

$$\frac{dy}{dx} = 1 - \frac{2}{\pi} \cos^2(x - y) \Rightarrow \left. \frac{dy}{dx} \right|_{\left(\frac{\pi}{2}, \frac{\pi}{4}\right)} = \frac{\pi - 1}{\pi}$$

$$\text{slope of tangent line} = m = \frac{\pi - 1}{\pi}, \text{ slope of normal line} = m' = -\frac{\pi}{\pi - 1}$$

$$\text{Equation of tangent line: } y - \frac{\pi}{4} = \frac{\pi - 1}{\pi} \left(x - \frac{\pi}{2}\right), \text{ Equation of normal line: } y - \frac{\pi}{4} = -\frac{\pi}{\pi - 1} \left(x - \frac{\pi}{2}\right)$$

c)

$$\frac{dy}{dx} = x^4 + x^3 + 9x^2 + 20x + ae^{ax}$$

$$\frac{d^2y}{dx^2} = 4x^3 + 3x^2 + 18x + 20 + a^2e^{ax}$$

$$\frac{d^3y}{dx^3} = 12x^2 + 6x + 18 + a^3e^{ax}$$

$$\frac{d^4y}{dx^4} = 24x + 6 + a^4e^{ax}$$

$$\frac{d^5y}{dx^5} = 24 + a^5e^{ax}$$

$$\frac{d^6y}{dx^6} = a^6e^{ax}$$

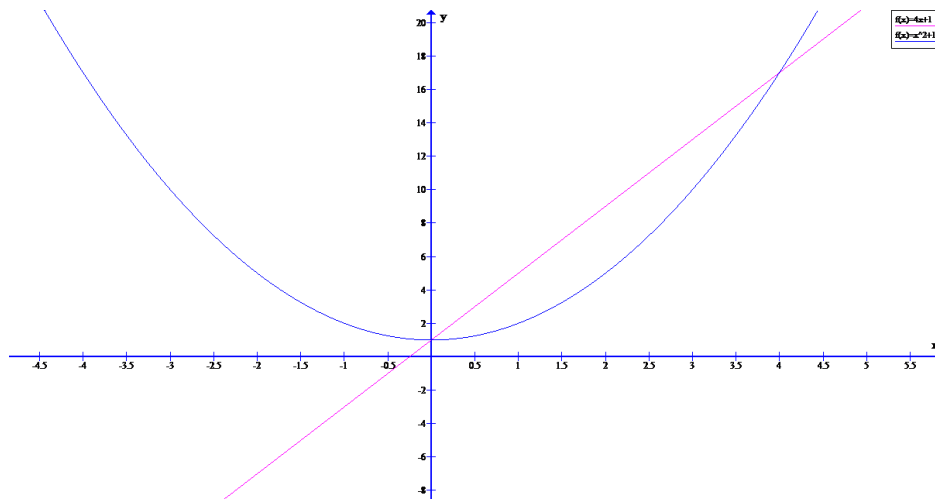
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$$\frac{d^ny}{dx^n} = a^ne^{ax}$$

Question: 2.

a)



Therefore, the points of intersection are (0, 1) and (4, 17).

$$\begin{aligned} \text{The area of bounded region} &= \int_0^4 [(4x+1) - (x^2+1)] dx \\ &= \int_0^4 (4x - x^2) dx \\ &= \frac{32}{3} \text{ square units} \end{aligned}$$

b)(i)

$$\text{By partial fractions: } \frac{x}{x^4-1} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1}$$

$$\Rightarrow x = A(x-1)(x^2+1) + B(x+1)(x^2+1) + (Cx+D)(x^2-1)$$

$$\Rightarrow A+B+C=0, -A+B+D=0, A+B-C=1, -A+B-D=0.$$

$$\text{After simplification } A = \frac{1}{4}, B = \frac{1}{4}, C = -\frac{1}{2}, D = 0. \text{ Hence } \int \frac{x}{x^4-1} dx = \ln \sqrt[4]{\frac{x^2-1}{x^2+1}} + C$$

(ii)

Putting $1-x=u^2 \Rightarrow x=1-u^2 \Rightarrow dx=-2udu$. Therefore,

$$\begin{aligned} \int_{-3}^{-1} \frac{x^2}{\sqrt{1-x}} dx &= -2 \int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} (1-u^2)^2 du = 2 \int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} (1-u^2)^2 du = 2 \int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} (1-2u^2+u^4) du \\ &= \frac{92-44\sqrt{2}}{15} \end{aligned}$$

Question: 3.

a)

Let $a_n = \frac{1.5.9...(4n-3)}{2.4.6...(2n)} (x+6)^n$. Then applying ratio test: $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{(4n-1)(4n+1)}{(2n+1)(2n+2)} |x+6|$
 $= 4|x+6|$

Therefore, by ratio test the series converges if $4|x+6| < 1 \Rightarrow |x+6| < \frac{1}{4} \Rightarrow -\frac{1}{4} < x+6 < \frac{1}{4}$

$\Rightarrow -\frac{25}{4} < x < -\frac{23}{4}$. Hence center, radius of convergence and interval of convergence are $-6, \frac{1}{4}$ and $(-\frac{25}{4}, -\frac{23}{4})$ respectively.

b)

Applying root test: $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{n+2}{3n+1}\right)^n} = \lim_{n \rightarrow \infty} \frac{n+2}{3n+1} = \frac{1}{3} < 1$. It implies that the given series converges absolutely.

Question: 4.

(i)

$$\begin{aligned} \int_0^\infty x^2 e^{-x^3} dx &= \lim_{b \rightarrow \infty} \int_0^b x^2 e^{-x^3} dx = -\frac{1}{3} \lim_{b \rightarrow \infty} \int_0^b -3x^2 e^{-x^3} dx = -\frac{1}{3} \lim_{b \rightarrow \infty} \left| e^{-x^3} \right|_0^b \\ &= -\frac{1}{3} \left(\lim_{b \rightarrow \infty} e^{-b^3} - \lim_{b \rightarrow \infty} e^0 \right) = -\frac{1}{3} (0-1) = \frac{1}{3} \end{aligned}$$

(ii)

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{1-\cos(2/x)}{\tan(3/x)} &= \lim_{x \rightarrow \infty} \frac{-\frac{2}{x^2} \sin(2/x)}{-\frac{3}{x^2} \sec^2(3/x)} \quad (\text{By L. H\^opital Rule}) \\ &= \frac{2}{3} \lim_{x \rightarrow \infty} \frac{\sin(2/x)}{\sec^2(3/x)} = \frac{2}{3} \cdot \frac{0}{1} = 0 \end{aligned}$$