



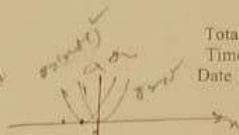
COMSATS University Islamabad
Department of Computer Science
Midterm Exam Spring 2024

Program : BCS-III
Teacher: Dr. M. Yousaf
Subject: Calculus and Analytical Geometry

Total Marks : 25
Time Allowed: 1 hr & 20 min
Date 17-04-2024

Attempt all questions

Q.1 Sketch the graph of the equation $y = -2(x+1)^2 - 3$ by making translating, reflecting, and compressing/stretching the graph of $y = x^2$ appropriately.



Q.2 An automobile is driven down a straight highway such that after $0 \leq t \leq 12$ seconds its $s = 4.5t^2$ feet from its initial position. Find the average velocity of the car over the time interval $[0, 12]$ and the instantaneous velocity of the car at $t = 6$.

$$v_{av} = \frac{4.5(12)^2 - 4.5(0)^2}{12 - 0} = 90 \text{ ft/sec}$$

Q.3 A spherical balloon is to be deflated so that its radius decreases at a constant rate of 15 cm/min. At what rate must air be removed when the radius is 9 cm?

$$v = \frac{4}{3}\pi r^3$$

Q.4 Find the limit using L-Hopital rule $\lim_{x \rightarrow 0} \left(\frac{e^x - 1}{\sin x} \right)$ and determine whether $f(x) = \frac{\ln x}{x}$ has a removable discontinuity at $x = 0$.

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x} = \frac{e^0 - 1}{\sin 0} = \frac{0}{0}$$

Q.5 Show that

$$f(x) = \begin{cases} x \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

is continuous at $x = 0$. Determine whether f is differentiable at $x = 0$. If so, find the value of the derivative there. Sketch the graph of f near $x = 0$.

$$-|x| \leq x \sin \frac{1}{x} \leq |x|$$

$$\lim_{x \rightarrow 0} -|x| = \lim_{x \rightarrow 0} |x| = 0$$

$$\Rightarrow \lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0 = f(0)$$

f is continuous at $x = 0$

$$f'(0) = \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{h \sin(1/h) - 0}{h} = \lim_{h \rightarrow 0} \sin(1/h) \text{ does not exist}$$

