

**Final Examination**

|                      |                                                                   |                |                 |               |            |
|----------------------|-------------------------------------------------------------------|----------------|-----------------|---------------|------------|
| Course Title:        | Calculus and Analytic Geometry                                    | Course Code:   | MTH104          | Credit Hours: | 3(3)       |
| Course Instructor/s: | Dr. Mohsan Hassan, Dr. Aqeel, Dr. Shumaila, Dr. Tahir, Ms. Shazia | Program Name:  | BSE/BCS/BCE/BEE |               |            |
| Semester:            | 1 <sup>st</sup>                                                   | Batch:         |                 | Section:      | C          |
| Time Allowed:        | 3 Hours                                                           | Maximum Marks: | 100             | Date:         | 11-01-2018 |
| Student's Name:      |                                                                   | Reg. No.       |                 |               |            |

**Important Instructions / Guidelines:**

- Attempt all questions.
- Do all parts of a question in a sequence.
- Do not write anything on question paper except your name and registration number
- Calculator sharing is not allowed.

**Question 1:** Solve the inequality and express the solution in term of interval

$$|2x - 7| > 3 \quad (-\infty, 2) \cup (5, +\infty) \quad (10)/10$$

**Question 2:** For what value of  $a$

$$f(x) = \begin{cases} a^2x - 2a, & x \geq 2 \\ 12, & x < 2 \end{cases} \quad a = 3, -2 \text{ continuous} \quad (10)/10$$

is continuous at  $x = 2$ .

**Question 3:** Find an equation for the tangent and normal lines to the parabola  $y = x^2$  at the point  $P(1, 1)$ .

$$\frac{y_2 - y_1 \cos(\theta)}{\log + u \cos(\theta)}$$

**Question 4:** Use implicit differentiation to find  $\frac{d^2y}{dx^2}$  if  $5y^2 + \sin(xy) = x^2$ .

**Question 5:** Evaluate the following integrals

$$\frac{d^2n}{d\theta^2}$$

$$(a) \int \frac{1}{x^3} \sqrt{\frac{x^2-1}{x^2}} dx \quad \int \sin^3 \theta \cos^2 \theta d\theta$$

$$(b) \int x^3 \cos(x) dx \quad n^3 \sin n + 3n^2 \cos n - 6n \sin n - 6 \cos n$$

**Question 6:** (a) Find  $dy/dx$  by chain rule if  $y = f(u)$  and  $u = g(x)$  are given as

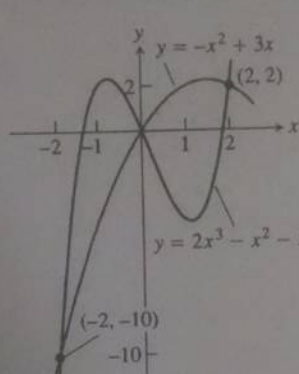
$$y = -\sec u, u = x^2 + 7x$$

(b) Write the function in form  $y = f(u)$  and  $u = g(x)$ . Then find  $dy/dx$  by chain rule if

$$u^3 \left( \frac{1}{n} + 1 + \frac{1}{n^2} \right)$$

$$\int_0^2 (-2x^3 + 8x) \quad y = \left( \frac{x^2}{8} + x - \frac{1}{x} \right)^4$$

**Question 7:** Find the area between curves as shown in Fig.



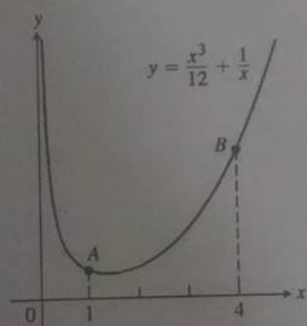
$$a=0$$

$$b=2$$

$$(0-2) \Rightarrow$$

$$\Rightarrow 8 \text{ sq. units}$$

**Question 8:** Find the arc length of the graph of  $f(x) = \frac{x^3}{12} + \frac{1}{x}$ ,  $1 \leq x \leq 4$ .



$$\int \sqrt{1 + \left( \frac{dy}{dx} \right)^2} dx$$

du

$$\frac{u^2 - 4}{4u^2}$$

$$\frac{(u^2 - 2)(u^2 + 2)}{(2u)^2} \cdot \frac{1}{u}$$

**Question 9:** Does the sequence whose nth term is  $a_n = \sqrt[n]{n^2}$  converge? If so, find  $\lim_{n \rightarrow \infty} (a_n)$ .

**Question 10:** Investigate the convergence of the following series

(a)  $\sum_{n=1}^{\infty} \frac{(2n)!}{n! n!}$  by ratio test,  $4 > 1$  divergent.

$$2(n+1) = 2n+2$$

(b)  $\sum_{n=1}^{\infty} \left( \frac{1}{1+n} \right)^n$  by root test,  $= 1$  no conclusion.

$$\sec x \tan x$$

Best of Luck

$$\frac{1}{1+y}$$

$$\frac{1}{1+\frac{1}{n}}$$