



COMSATS University (CUI)
Islamabad Campus
Department of Mathematics
Terminal Exam. (Fall 2022)

Program: BSM-III

Instructor: Dr. Manshoor Ahmed

Date: 30-01-2023

Subject: Calculus-III (MTH 203)

Total Marks: 50

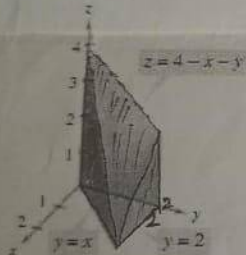
Time: 9:00-12:00

- Q1. Find the projection of $\mathbf{u}$ onto $\mathbf{v}$ and the vector component of $\mathbf{u}$ orthogonal to $\mathbf{v}$ for the vectors $\mathbf{u} = 3\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = 7\mathbf{i} + \mathbf{j} - 2\mathbf{k}$
- Q2. Find the symmetric equation of the line through the point $(5, 0, -2)$ that is parallel to the planes $x - 4y + 2z = 0$ and $2x + 3y - z + 1 = 0$.

Q3. (a) Find parametric equations of the line tangent to the graph of $\mathbf{r}(t) = \ln t \mathbf{i} + e^{-t} \mathbf{j} + t^3 \mathbf{k}$; at the point where $t_0 = 2$.

(b): Find the spherical coordinates of the point that has rectangular coordinates $(x, y, z) = (1, -1, \sqrt{2})$.

Q4. Use the double integral to find the volume of the solid shown in the fig.



Q5. Use the spherical coordinates to find the volume of the solid within the sphere $x^2 + y^2 + z^2 = 9$, outside the cone $z = \sqrt{x^2 + y^2}$ and above the xy -plane.

Q6. Evaluate the integral by making an appropriate change of variables:

$\iint_R e^{(x+y)/(x-y)} dA$ where R is the trapezoidal region with vertices $(1, 0)$, $(2, 0)$, $(0, -2)$ and $(0, -1)$.

Q7. Find the mass of a thin wire shaped in the form of the helix

$$x = 3 \cos t, \quad y = 3 \sin t, \quad z = 4t, \quad 0 \leq t \leq \pi/2.$$

If the density function is $\delta = \frac{kx}{1+y^2}$ ($k > 0$).

Q8. Show that the integral is independent of the path, and use Fundamental Theorem of line integral to find its value. $\int_{(0,0)}^{(1, \frac{\pi}{2})} e^x \sin y dx + e^y \cos x dy$

Q9. Use the Green's Theorem to evaluate the line integral $\oint_C \ln(1+y)dx - \frac{xy}{1+y} dy$, where C is the triangle with vertices $(0,0)$, $(2,0)$, $(0,4)$.

Q10. Use Divergence theorem to find the flux of $\mathbf{F}$ across the surface σ with outward orientation. $\mathbf{F}(x,y,z) = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$, σ is the surface of the cylindrical solid bounded by $x^2 + y^2 = 4$, $z = 0$, and $z = 1$.



COMSATS University Islamabad (CUI)
Islamabad Campus
Department of Electrical Engineering

Final Term Exam EEE Multivariable Calculus

BEE, EEE & BCE

Marks: 50

Date: June 24, 2025

Note: Attempt all questions diligently.

Version 1

Time: 180 minutes

Instructor: Dr. Qudrat Khan

Q.1: (Marks 10) Find the point of intersection of the plane $x + 2y + z = 10$ and the paraboloid $z = x^2 + y^2$ that is closest to the origin. Also verify your answer. Clue: Use Lagrange multiplier method with two constraints.

Q. No.2 (Marks 2.5+2.5) Choose the limits of the integrals. Don't solve it further once the limits with proper order are displayed.

Find the volume of the solid given by

- Under the plane $x + 2y - z = 0$ and above the region bounded by $y = x$ and $y = x^4$.
- Enclosed by the paraboloid $z = x^2 + 3y^2$ and the plane $x = 0, y = 1, y = x$ and $z = 0$.

Q. No.3: (Marks 10) Find the center of mass of the solid of constant density that is bounded by parabolic cylinder $x = y^2$ and the planes $x = z, z = 0$ and $x = 2$.

Q. No.4: (Marks 10) Use Green's Theorem to evaluate the line integral $\oint_C (y + e^x)dx + (2x + \cos y^2)dy$ along the positively oriented curve C enclosed by the parabolas $y = x^2$ and $x = y^2$.

Q. No.5: (Marks 5) Find the equation of the tangent plane and normal line at the point $(-2, 1, -3)$ to the ellipsoid $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$

Q. No.6: (Marks 10) Evaluate $\oint_C y^2 dx + x dy$ where a). $C = C_1$ is the line segment from $(-5, -3)$ to $(0, 2)$ and b). $C = C_2$ is the arc of the parabola $x = 4 - y^2$ from $(-5, -3)$ to $(0, 2)$.

** Hold your horses and think wisely before taking down your answers.*

Handwritten notes:
 $\frac{20}{2} \times \frac{2}{4} P$
 $2 - 1 = 1$
 $(2y^2 + \cos(x^2))^2 - \frac{2}{3} y^3 (e^{y^2})$