COMSATS University, Islamabad (CUI)

Islamabad Campus

Fall 2024

Midterm Exam

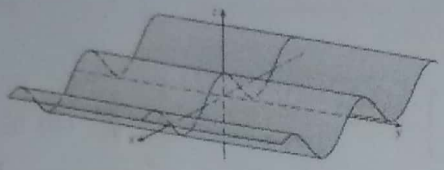
Class: BCS-R

Subject: Multivariable Calculus (MTH 105)

Instructor: Dr. Manshoor Ahmed

Date: 05-11- 2024, Time [11:30-12:50]

Q1. (i) Which of the following is the equation of straight line $x = a$ in polar coordinates:
(a) $r = a\cos\theta$ (b) $r = a\sin\theta$ (c) $r = a\sec\theta$ (d) $r = a\operatorname{cosec}\theta$.
(ii) The equation of the given the cylindrical surface



(a) $y = x^2$ (b) $z = \cos x$ (c) $z = x^2$ (d) $z = \sin x$

(iii) Which of the following point lie on the curve $r = a\cos\theta$ is
(a) $(-a, \pi)$. (b) (a, π) (c) $(a, \pi/2)$. (d) $(a, 0)$

(iv) If $f(x, y) = \sin(xy) + x^2 ln(y)$, then f_{yx} at $(0, \pi/2)$:

(a) 33 (b) 0 (c) 3 (d) 1

(v) The function $f(x, y) = x^2 + 2xy - y^3$
(a) Only one stationary point at (0,0). (b) Two stationary point at (0,0) and (1/6, 1/3).
(c) Two stationary point at (0,0) and (1/6, 1/3). (d) No stationary point.

Q2. Sketch and find the area of the region that is outside the cardioid $r = 2 - 2\cos\theta$ and inside the circle $r = 4$.

Q3. Find the eccentricity and the distance from the pole to the directrix and sketch the plot in polar coordinates.

$$r = \frac{4}{3 - 2\cos\theta}.$$

Q4. Solve the following problem by using Lagrange Multipliers. Find the point on the circle $x^2 + y^2 = 45$ that are closest and farthest from $(1,2)$.

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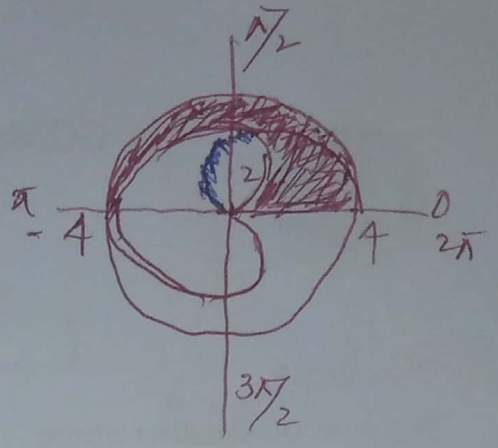
Q#2

Points of intersection of two curves.



$r = 2 - 2\cos\theta$ & $r = 4$

$2 - 2\cos\theta = 4$

$1 - \cos\theta = 2$

$\cos\theta = -1 \Rightarrow \theta = \cos^{-1}(-1) =$

$\theta = \pi$

∴ Required area.

$$A = 2\left[\frac{1}{2}\int_{0}^{\pi}\left[(4)^2 - (2-2\cos\theta)^2\right]d\theta\right]$$ 2× due to symmetry.

$$A = \int_0^{\pi} 16\, d\theta - \int_0^{\pi}(4 + 4\cos^2\theta - 8\cos\theta)\,d\theta$$

$$= 16[\theta]_0^{\pi} - 4\int_0^{\pi}(1 + \cos^2 - 2\cos\theta)\,d\theta$$

$$= 16\pi - 4\int_0^{\pi}\left(1 + \left(\frac{1+\cos 2\theta}{2}\right) - 2\cos\theta\right)d\theta$$

$$= 16\pi - 4\pi - \frac{4}{2}\left(\theta + \frac{\sin 2\theta}{2}\right)\Big|_0^{\pi} - 8\,|\sin\theta|_0^{\pi}$$

$$= 12\pi - 2\pi - 0 - 8(0-0)$$

$$= 8\pi.$$

x ——————— y

Q#3 Given equation is

$$r = \frac{4}{3 - 2\cos\theta}$$

$$r = \frac{4/3}{1 - \frac{2}{3}\cos\theta} = \frac{\frac{2}{3}(2)}{1 - \frac{2}{3}\cos\theta}$$

Here, $e = \frac{2}{3}$, $d = 2$, For the $0 < e < 1$.

∴ the given eq. represents ellipse.

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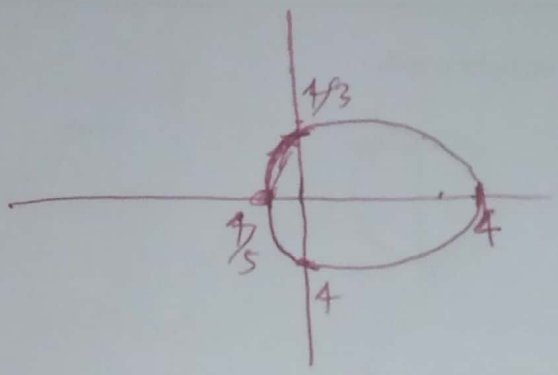
Table of value

θ	0	$\pi/2$	π	$3\pi/2$	2π
r	4	$4/3$	$4/5$	$4/3$	4



$\theta = 0$

$r = \frac{4}{3 - 2\cos 0} = 4$

$\theta = \pi/2$

$r = \frac{4}{3 - 2\cos \pi/2} = 4/3$

$\theta = \pi$

$r = \frac{4}{3 - 2\cos \pi} = \frac{4}{5}$

$\theta = 3\pi/2, r = \frac{4}{3 - 2\cos 3\pi/2}$

$r = \frac{4}{3}$

$\theta = \pi$

$r = \frac{4}{3 - \cos \pi} = 4$

Q #4 Let (x, y) be the point on the given circle.

Then distance of (x, y) from $(1, 2)$ is

$f(x,y) = D^2 = (x-1)^2 + (y-2)^2$

$f(x,y) = (x-1)^2 + (y-2)^2$

Subject to $g(x,y) = x^2 + y^2 - 45 \rightarrow (1)$

$\nabla f(x,y) = 2(x-1)\hat{i} + 2(y-2)\hat{j}$ $\nabla g = 2x\hat{i} + 2y\hat{j}$

Since $\nabla g \neq 0$ at the circle. $\therefore$ Extreme values exist at the points, where,

$\nabla f = \lambda \nabla g$

$\Rightarrow 2(x-1)\hat{i} + 2(y-2) = \lambda(2x\hat{i} + 2y)$

Comparison gives

$2(x-1) = \lambda(2x), \quad 2(y-2) = \lambda(2y)$

$\Rightarrow \frac{x-1}{x} = \lambda, \quad \frac{y-2}{y} = \lambda$

$\frac{x-1}{x} = \frac{y-2}{y} \Rightarrow y(x-1) = x(y-2)$

$xy - y = xy - 2x$

$y = 2x \rightarrow (ii)$

Using (ii) into (i)

$x^2 + (2x)^2 = 45 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

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When $x = +3$ then $y = 6$

When $x = -3$ then $y = -6$.

∴ Possible Points are.

$(3,6)$, $(-3,-6)$

As $f(3,6) = (3-1)^2 + (6-2)^2 = 20$

$f(-3,-6) = (-3-1)^2 + (-6-2)^2 = 20$.

Therefore, $(3,6)$ is closet and $(-3,-6)$ is farthest.