



COMSATS University Islamabad (CUI)
Islamabad Campus
Terminal Exam Spring 2022

Program: BEE-2 (A, B, C), BEEE

Subject: MTH-105, Multivariable Calculus

Date: June 28, 2022.

Total Marks: 50

Maximum time: 3 hrs.

Instructors: Dr. Javeria Nawaz Abbasi, Dr. Adeel Ahmed, Dr. Iffat Jabeen, Iqra Khan

Instructions for attempting paper:

1. Mobile phones and digital dairies will not be allowed in the examination hall.
2. Answer all questions.

Question #1: Suppose that a bee follows the trajectory

$$x = t - \cos t, y = 2 - 2\sin t, 0 \leq t \leq 10$$

- (a) At what times was the bee flying horizontally?
- (b) At what times was the bee flying vertically?

Question #2: Find the area of the region inside the circle $r = 2$ and to the right of the line $r = \sqrt{2} \sec \theta$.

Question #3: Find where the tangent line to the curve $r = e^{-2t} i + \cos t j + 3\sin t k$ at the point $(1, 1, 0)$ intersects the yz -plane.

Question #4: Show that the lines

$$L_1: x = -2 + t, y = 3 + 2t, z = 4 - t \quad \text{And}$$

$$L_2: x = 3 - t, y = 4 - 2t, z = t$$

are parallel and find an equation of the plane they determine.

Question #5: Use Lagrange multipliers to find the maximum and minimum values of

$$f(x, y, z) = 2x + y - 2z \quad \text{Subject to the constraint } x^2 + y^2 + z^2 = 4.$$

Question #6: Find a unit vector in the direction in which $f(x, y) = \cos(3x - y)$ decreases most rapidly at $P\left(\frac{\pi}{6}, \frac{\pi}{4}\right)$, and find the rate of change of f at P in that direction.

Question #7: Find the surface area of the portion of the curve $r(u, v) = u \cos v i + u \sin v j + u^2 k$ for which $1 \leq u \leq 2, 0 \leq v \leq 2\pi$.

Question #8: Find the volume of the solid in the first octant bounded above by the paraboloid $z = x^2 + 3y^2$ below by the plane $z = 0$, and laterally by $y = x^2$ and $y = x$.

Question #9: Evaluate the triple integral by changing to cylindrical coordinates.

$$\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} e^{-(x^2+y^2+z^2)^{3/2}} dz dy dx$$

Question #10: Evaluate the line integral along the curve C

$$\oint_C (x^2 - y^2) dx + x dy$$

$$C: x = t^{1/3}, y = t, -1 \leq t \leq 1.$$

Good Luck