



COMSATS University Islamabad (CUI)

Islamabad Campus

Terminal Exam Fall 2023

(Subjective part)

Department: Computer Sciences

Semester: 4TH

Subject: Linear Algebra

Instructors:

Dr. Bilal Ashraf

Dr. Adeel Ahmad

Dr. Shagufta Yasmeen

Dr. Nadia Riaz

Ms. Sharmeen

Total Marks: 40

Time Allowed: 2½ hours

Date: 6th January, 2024

1. Find the bases for ColA, RowA, and NulA. Also list rankA and dim NulA. (5)

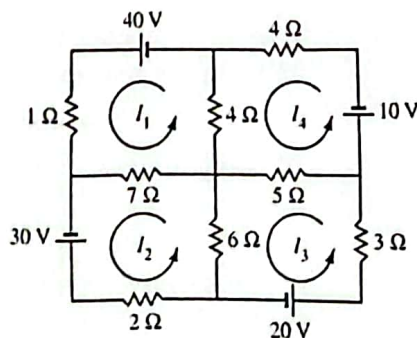
$$A = \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$$

2. Use the Gram-Schmidt process to produce an orthogonal basis for the column space of the matrix given in question 1. (5)

3. Diagonalize the given matrix, if possible (5)

$$\begin{bmatrix} 0 & -1 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

4. Write a matrix equation that determines the loop currents of the following circuit. Also solve the system for loop currents. (5)



(P.T.O)

5. Find the LU factorization of the following matrix:

(5)

$$\begin{bmatrix} 1 & 3 & -5 & -3 \\ -1 & -5 & 8 & 4 \\ 4 & 2 & -5 & -7 \\ -2 & -4 & 7 & 5 \end{bmatrix}$$

6. Answer the following short questions:

(3x5)

i. The first four Laguerre polynomials are $1, 1-t, 2-4t+t^2, 6-18t+9t^2-t^3$. Show that these polynomials form a basis of P_3 .

ii. Consider a unit close ball/sphere $H = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x^2 + y^2 + z^2 \leq 1 \right\}$. Show that H is not

a subspace of $\mathbb{R}^3$.

iii. Find the closest point to y in the subspace W spanned by v_1 and v_2 .

$$y = \begin{bmatrix} 3 \\ 1 \\ 5 \\ 1 \end{bmatrix}, v_1 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

iv. Show that the equation of the line in $\mathbb{R}^2$ through distinct points (x_1, y_1) and (x_2, y_2) can be written as

$$\det \begin{bmatrix} 1 & x & y \\ 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \end{bmatrix} = 0$$

v. Without calculation, find one eigenvalue and two linearly independent eigenvectors of A . Justify your answer.

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

Best of luck



COMSATS University Islamabad (CUI)
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Terminal Exam Fall 2023
(Objective part)

Department: Computer Sciences
Semester: 4th
Subject: Linear Algebra

Total Marks: 10
Time Allowed: First 30 mins

Instructors: Dr. Bilal Ashraf
Dr. Adeel Ahmad
Dr. Shagufta Yasmeen
Dr. Nadia Riaz
Ms. Sharmeen

Name: _____ Registration #: _____

Do as directed:

1. Columns of every invertible $n \times n$ matrix form the basis for $\mathbb{R}^n$. True or false? Give one liner justification for your answer.

2. A linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$ with invertible standard matrix of transformation is always one-to-one and onto $\mathbb{R}^n$ transformation. True or false? Give one liner justification for your answer.

3. If A is an $n \times n$ invertible matrix then Null space of A is $\mathbb{R}^n$. True or false? Give one liner justification for your answer.

4. If y is in $W = \text{Span}\{u_1, \dots, u_p\}$ then $\text{proj}_W y =$ _____.
a. $y - \hat{y}$ b. $\hat{y}$ c. y d. None of above
5. If W is a subspace of $\mathbb{R}^n$ and if v is in both W and $W^\perp$, then v must be _____.
a. the unit vector b. a vector in basis of W
c. the zero-vector d. a vector in basis of $W^\perp$
6. The following statement is incorrect "A is diagonalizable if and only if A has n eigenvalues, counting multiplicities". Write the correct version of this statement.

(P.T.O)

7. If $A = \begin{bmatrix} 1 & 5 & 8 \\ 0 & 0 & 7 \\ 0 & 0 & -2 \end{bmatrix}$, then

- a. A is diagonalizable b. A is singular c. both a and b d. neither a nor b

8. Two different matrices A and B have same characteristic equations and hence same eigenvalues if they are _____ matrices.

9. If β is standard basis of the vector space V and if x is in V then β -coordinate of x , $[x]_{\beta} =$ _____.

10. Standard matrix of transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ that is a vertical shear transformation that maps e_1 into $e_1 - 3e_2$ but leave e_2 unchanged is

- a. $\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ d. $\begin{bmatrix} -3 & 0 \\ 1 & 1 \end{bmatrix}$