



# COMSATS UNIVERSITY ISLAMABAD

Mid Exam (Fall 2023)

Course: MTH 231 – Linear Algebra  
Class: BCS-4B  
Instructor: Dr. Adeel Ahmad

November 13, 2023  
Time Allowed: 1½ hour  
Maximum Marks: 25

Instructions: Answer ALL questions. Only scientific (non-graphic, non-programmable) calculators are allowed. Use of mobile phones, digital diaries etc. is not allowed during the exam.

1. Find the LU factorization of the following matrix: (5)

$$\begin{bmatrix} 2 & -3 & 4 \\ -4 & 8 & -7 \\ 6 & -5 & 14 \\ -6 & 9 & -12 \\ 8 & -6 & 19 \end{bmatrix}$$

$$\begin{array}{l} R_2 - 4R_1 \\ 2R_1 \\ \hline 0 \quad 2 \quad 1 \end{array} \quad \begin{array}{l} R_3 \\ R_4 \\ R_5 \end{array} \quad \begin{array}{l} 0 \quad 1 \quad 0 \\ 0 \quad 1 \quad 1 \\ 0 \quad 1 \quad 1 \end{array}$$

2. Let

$$A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$$

- Determine the linear dependence/independence of the columns of A.
- Determine if the columns of A span the space  $\mathbb{R}^3$ .
- Determine if the transformation  $x \mapsto Ax$  is one-to-one and onto.

3. Is  $b$  in the range of the linear transformation  $x \mapsto Ax$ ? Why or why not?

$$A = \begin{bmatrix} 1 & -3 & 5 & -5 \\ 0 & 1 & -3 & 5 \\ 2 & -4 & 4 & -4 \end{bmatrix}, \quad b = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{array}{l} \text{Consistent} \\ b \text{ is in range} \\ R_3 - 2R_1 \\ R_1 \pm 1 \\ \hline 1 \quad -2 \quad 1 \\ 0 \quad 1 \quad -5 \\ 0 \quad 5 \quad 8 \end{array}$$

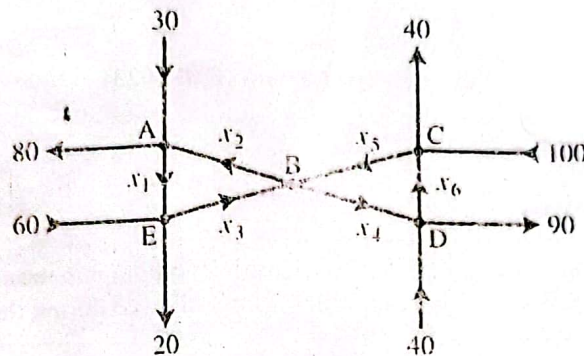
4. Assuming that the flow must be in the directions indicated. Find the general flow pattern of the network shown in the figure i.e. write equations that describe the flow.

$$\begin{array}{r} 2R_1 \\ -R_3 \\ \hline 0 \quad -2 \quad 6 \quad -6 \quad -2 \end{array} \quad \begin{array}{l} 2 \\ -4 \\ 4 \\ 4 \\ 0 \end{array}$$

$$\begin{array}{l} R_1 \\ R_3 \\ \hline 1 \quad 0 \quad 3 \\ \pm 1 \quad \pm 2 \quad 1 \\ 0 \quad 2 \quad 2 \end{array}$$

(P.T.O)

$$\begin{array}{l} R_2 \\ R_3 \\ \hline 0 \quad 1 \quad -3 \quad 5 \quad 1 \\ 0 \quad -1 \quad 3 \quad -3 \quad -1 \\ 0 \quad 0 \quad 0 \quad 2 \quad 0 \end{array}$$



$$x_1 + 80 = x_2 + 30$$

$$x_1 - x_2 = 30 - 80$$

$$A) x_1 - x_2 = -50$$

$$B) x_3 + x_5 = x_2 + x_1$$

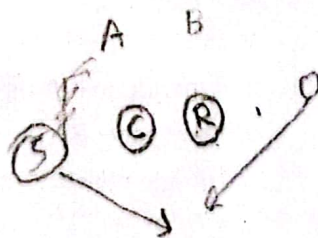
$$C) x_5 - x_6 = 60$$

$$30 + 40 + 100 + 60 = 40 + 20 + 90 + 40$$

5. Do as directed. Justify each answer.

(5)

- The equation  $\mathbf{x} = \mathbf{p} + t\mathbf{v}$  describes a line through  $\mathbf{v}$  parallel to  $\mathbf{p}$ . (True/False)
- If one row in an echelon form of an augmented matrix is  $[0 \ 0 \ 0 \ 5 \ 0]$ , then the associated linear system is inconsistent. (True/False)
- How many rows does  $\mathbf{B}$  have if  $\mathbf{BC}$  is a  $5 \times 4$  matrix?
- If  $\mathbf{A}$  is a  $6 \times 5$  matrix, the linear transformation  $\mathbf{x} \mapsto \mathbf{Ax}$  map \_\_\_\_\_ onto \_\_\_\_\_.
- If  $\mathbf{A}$  is an  $m \times n$  matrix and the equation  $\mathbf{Ax} = \mathbf{b}$  is consistent for every  $\mathbf{b}$  in  $\mathbb{R}^m$ , then  $\mathbf{A}$  has \_\_\_\_\_ pivot columns.



BC  
5x4