



COMSATS University Islamabad (CUI)
Islamabad Campus

Terminal Exam FALL 2024

Program: BCS, BAI, ELE
Semester: 4th
Subject: Linear Algebra
Date: January 17, 2025

Total Marks: 50
Time Allowed: 3 hours
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1. Find the bases for $\text{Col}A$, $\text{Row}A$, and $\text{Nul}A$. Also list $\text{rank}A$ and $\dim \text{Nul}A$. (5)

$$\begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$$

2. The first four Laguerre polynomials are $1, 1-t, 2-4t+t^2, 6-18t+9t^2-t^3$. Show that these polynomials form a basis of P_3 . (5)

3. Find the closest point to y in the subspace W spanned by v_1 and v_2 . (5)

$$y = \begin{bmatrix} 3 \\ 1 \\ 5 \\ 1 \end{bmatrix}, v_1 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

4. Let $\{x_1, x_2, x_3\}$ be the basis for the subspace W of $\mathbb{R}^4$. With the help of Gram-Schmidt process construct the orthogonal basis for W . (5)

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

5. Diagonalize the given matrix with the real eigenvalues $\lambda = 1, 5$. (5)

$$\begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$$

(P.T.O)

6. Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear transformation determined by the matrix: (5)

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

Let S be the unit ball, whose bounding surface has the equation $x_1^2 + x_2^2 + x_3^2 = 1$. Determine the volume of the region bounded by $T(S)$.

7. Find the inverses of the following matrix using the row operations. (5)

$$\begin{bmatrix} 1 & 2 & -1 \\ -4 & -7 & 3 \\ -2 & -6 & 4 \end{bmatrix}$$

8. Find the LU factorization of the following matrix: (5)

$$\begin{bmatrix} 1 & 3 & -5 & -3 \\ -1 & -5 & 8 & 4 \\ 4 & 2 & -5 & -7 \\ -2 & -4 & 7 & 5 \end{bmatrix}$$

9. Consider the standard matrix of transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^4$ is: (5)

$$\begin{bmatrix} 3 & 4 & 9 \\ -2 & -7 & 7 \\ 1 & 2 & -2 \\ 0 & 2 & -6 \end{bmatrix}$$

Determine whether the transformation T is one-to-one and $\mathbb{R}^3$ onto $\mathbb{R}^4$.

10. Determine whether the given system of linear equations is consistent. (5)

$$\begin{aligned} x_1 - 6x_2 &= 5 \\ x_2 - 4x_3 + x_4 &= 0 \\ -x_1 + 6x_2 + x_3 + 5x_4 &= 3 \end{aligned}$$

Best of luck