



COMSATS University Islamabad (CUI)

Islamabad Campus
Department of Computer Sciences
Terminal Exam Spring 2023

Programs: BCS/BSSE

Time Allowed: 3 Hours

Instructor: Dr. Saqib Zia/Dr. Iffat Jabeen/Ms. Moneeza Batool

Total Marks: 50

Exam Date: June 13, 2023

Attempt all questions. All questions carry equal marks.

Q1. Find an orthogonal basis for the column space of matrix given below.

$$A = \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$$

Q2. Let $y = \begin{bmatrix} -1 \\ 4 \\ 5 \end{bmatrix}$, $u_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $u_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$. Decompose y into sum of two vectors, one is in the $W = \text{span}\{u_1, u_2\}$ and other is orthogonal to W .

Q3. Diagonalize the matrix $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$, if possible.

Q4. Is $\lambda = 1$ is an eigen value of $\begin{bmatrix} 4 & -2 & 3 \\ 0 & -1 & 3 \\ -1 & 2 & -2 \end{bmatrix}$? Why or why not?

Q5. For the matrix $A = \begin{bmatrix} 3 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & 7 \end{bmatrix}$. Define transformation $T = \mathbb{R}^3 \rightarrow \mathbb{R}$ by $T(x) = \text{determinant of the matrix } A \text{ is obtained by replacing the first column by } \vec{x} = (x_1, x_2, x_3)^T$. Find the image of $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$. Is the transformation is linear?

Q6. Let B be the basis of $\mathbb{P}_3$ consisting of Hermite polynomials $1, 2t, -2 + 4t^2$ and $-12t + 8t^3$, and let $P(t) = -1 + 8t^2 + 8t^3$. Find the coordinate vector of P relative to B .

Q7. Find an LU of the matrix A and solve the equation $AX = b$ by using the LU factorization.

$$A = \begin{bmatrix} 2 & -6 & 4 \\ -4 & 8 & 0 \\ 0 & -4 & 6 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix}$$

Q8. Find h in the matrix $A = \begin{bmatrix} 4 & 2 & 3 & 3 \\ 0 & 2 & h & 3 \\ 0 & 0 & 4 & 14 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ such that the eigenspace for $\lambda = 4$ is two-dimensional.

Q.9. Let $w = \begin{bmatrix} 4 \\ -1 \\ 4 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 5 & -1 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{bmatrix}$. Determine if w is in $\text{Col } A$. Is w in $\text{Nul } A$?

Q.10. Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, and $H = \left\{ \begin{bmatrix} s \\ s \\ 0 \end{bmatrix} : s \text{ in } \mathbb{R} \right\}$. Is $\{v_1, v_2\}$ a basis for H ?