



# COMSATS University Islamabad (CUI)

Islamabad Campus  
Midterm Examination, Spring 2024

April 16, 2024

Time Allowed: 90 minutes

Total Marks: 25

Course: MTH 231 - Linear Algebra

Level: BCT-IV, BDS-IV, BEI-IV, BCS-IV, BSE-III

Instructors: Dr. Tanvir Akber Kiani

Dr. Qazi Muhammad Zaigham Zia

Dr. Adeel Ahmed

## Instructions:

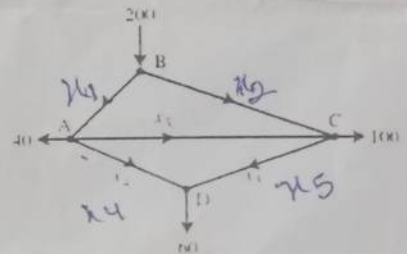
Answer ALL questions. All questions carry equal marks. Show all the work. Answers without explanations will get partial credits only.

Question No. 01: [5 points] Utilize your answer sheet to provide responses as either true or false.

- A basic variable in a linear system is a variable that corresponds to a pivot column in the coefficient matrix. *True*
- If the coefficient matrix  $A$  has a pivot position in every row, then the equation  $Ax = b$  is inconsistent. *False*
- If  $A$  is a  $3 \times 5$  matrix and  $T$  is a transformation defined by  $T(x) = Ax$  then the domain of  $T$  is  $\mathbb{R}^3$ . *False*
- LU factorization of a matrix  $A$  is possible only if the matrix  $A$  is a square matrix. *False*
- If  $A$  and  $B$  are  $n \times n$  and invertible, then  $A^{-1}B^{-1}$  is the inverse of  $AB$ . *True*

Question No. 02: [5 points]

- Find the general traffic pattern of the freeway network shown in the figure. (Flow rates are in Cars/minutes).
- Describe the general traffic pattern when the road whose flow is  $x_4$  is closed.
- When  $x_4 = 0$ , what is the minimum value of  $x_1$ ?



Question No. 03: [5 points] Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear transformation that first performs a horizontal shear that transforms  $e_2$  into  $e_2 + 2e_1$  (leaving  $e_1$  unchanged) and reflects points through the line  $x_2 = -x_1$ . Find the standard matrix of  $T$ .

Question No. 04: [5 points] Find an LU factorization of the following matrix

$$A = \begin{bmatrix} 1 & -5 & -3 \\ -1 & 8 & 4 \\ 4 & -5 & -7 \\ -2 & 7 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}$$

Question No. 05: [5 points] Determine if the vectors  $(4, 6, -7)$ ,  $(-7, 0, 2)$  and  $(-3, -5, 6)$  are linearly independent or linearly dependent.