



COMSATS University Islamabad

Islamabad Campus

Terminal Examination, Spring 2025

July 7, 2025

Course: MTH 231 - Linear Algebra

Class: BCS/BAI/BDS

Instructors: Dr. Adeel Ahmad and Dr. Fahad Munir Abbasi

Time Allowed: 3 hours

Total Marks: 50

Instructions: Answer ALL questions. All questions carry equal marks. Answers without explanations will get partial credits only.

Question No. 01: Diagonalize the following matrix (if possible):

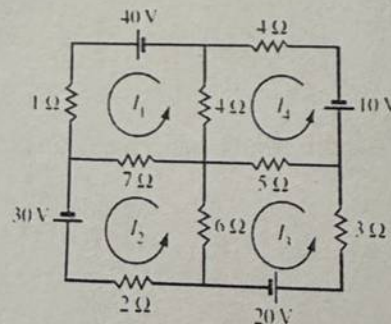
$$\begin{bmatrix} 2 & -2 & -2 \\ 3 & -3 & -2 \\ 2 & -2 & -2 \end{bmatrix}$$

Question No. 02: Let $\{x_1, x_2, x_3\}$ is a basis for a subspace W of $\mathbb{R}^4$. With the help of Gram-Schmidt process construct an orthogonal basis for W :

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} -3/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ -2/3 \\ -2/3 \\ 1/3 \end{bmatrix}$$

Question No. 03: Write a matrix equation that determines the loop currents of following circuit. Also solve the system for loop currents:



Question No. 04: Determine if the given linear transformation is (a) one to one and (b) onto.

$$T(x_1, x_2, x_3) = (x_1 - 5x_2 + 4x_3, x_2 - 6x_3)$$

Question No. 05: Find the LU factorization for the following matrix:

$$\begin{bmatrix} 2 & -4 & -2 & 3 \\ 6 & -9 & -5 & 8 \\ 2 & -7 & -3 & 9 \\ 4 & -2 & -2 & -1 \\ -6 & 3 & 3 & 4 \end{bmatrix}$$

Question No. 06: Find the area of parallelogram whose vertices are given as:
 $(-1, 0), (0, 5), (1, -4), (2, 1)$.

Question No. 07: Let W be a set of all vectors of the form $\begin{bmatrix} s+3t \\ s-t \\ 2s-t \\ 4t \end{bmatrix}$, show that W is a subspace of

$$\mathbb{R}^4 \quad \begin{bmatrix} 1 & 3 \\ 0 & -4 \\ 0 & -7 \\ 0 & 4 \end{bmatrix}$$

Question No. 08: Find the determinants:

i.

$$\begin{vmatrix} 3 & 5 & -8 & 4 \\ 0 & -2 & 3 & -7 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 2 \end{vmatrix}$$

-12

ii.

$$\begin{vmatrix} 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \end{vmatrix}$$

1

Question No. 09: Find the basis for column space and null space of A

$$A = \begin{bmatrix} 1 & 4 & 8 & -3 & -7 \\ -1 & 2 & 7 & 3 & 4 \\ -2 & 2 & 9 & 5 & 5 \end{bmatrix}$$

Question No. 10: Find the orthogonal complement of column space of matrix A given in Question 09.

Best of Luck