



COMSATS University Islamabad (CUI)

Department of Computer Science

Final Examination Fall 2023

Program: BCS-5A,B/BDS-4A

Subject: Differential Equations MTH242

Name of Instructor: Dr.Saqib Zia/ Dr. Zahoor Shah

Total Marks: 50

Date: January 11, 2024

Instructions for attempting paper:

1. Mobile phones are not allowed in the examination hall.
2. Sharing of anything is forbidden.
3. Attempt all questions.

Q1. Determine the singular points of the given differential equation. Classify each singular point as regular or irregular.

$$(x^2 + x - 6)y'' + (x + 3)y' + (x - 2)y = 0.$$

Q2. Use the power series method to solve the following differential equation around $x = 0$.

$$(x^2 + 1)y'' + 2xy' = 0$$

Q3. The function $y_1 = \ln x$ is a solution of $xy'' + y' = 0$. Find the general solution of the differential equation on the interval $(0, \infty)$.

Q4. Solve the given initial-value problem.

$$\begin{aligned} x^2 y'' - 4xy' + 6y &= 0 \\ y(-2) &= 8, \quad y'(-2) = 0 \end{aligned}$$

Q5. Solve the following differential equation by the method of undetermined coefficients:

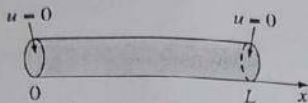
$$y'' - y' + \frac{1}{4}y = 3 + e^{\frac{x}{2}}.$$

Q6. The population of a town grows at a rate proportional to the population present at time t . The initial population of 500 increases by 15% in 10 years. What will be the population in 30 years? How fast is the population growing at $t = 30$.

Q7. Determine whether the given differential equation is exact, if it exact solve it.

$$\left(x^2 y^3 + \frac{1}{1+x^2}\right) \frac{dx}{dy} + x^3 y^2 = 0$$

Q8. Consider a thin rod of length L with an initial temperature $f(x)$ throughout. Assumed that the ends of the rod are held at zero temperature for all time $t > 0$ and the temperature of the rod is determined by the boundary-value problem.



$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \quad 0 < x < L, \quad t > 0$$

$$u(0, t) = 0, \quad u(L, t) = 0, \quad t > 0$$

$$u(x, 0) = f(x), \quad 0 < x < L.$$

Solve the boundary value problem for $u(x, t)$.

Q.9. Describe the physical meaning of given Partial Differential equation,

$$\alpha^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < L, \quad t > 0$$

$$u(0, t) = 0, \quad u(L, t) = 0, \quad t > 0$$

$$u(x, 0) = f(x), \quad \frac{\partial u}{\partial t} \Big|_{t=0} = g(x), \quad 0 < x < L.$$

Q.10. A semi-infinite plate coincides with the region defined by $0 \leq x \leq \pi, y \geq 0$. The left end is held at temperature e^{-y} , and the right end is held at temperature at 100 for $0 < y \leq 1$ and the temperature zero for $y > 1$. The bottom of the plate is held at temperature $f(x)$. Write the related differential equation and boundary conditions. (No need to solve).