



COMSATS University Islamabad (CUI)

Park Road, Tarlai Kalan, Islamabad 45550, Pakistan

Department of Mathematics

Terminal Examination, Spring 2025

Course: MTH242 - Differential Equations

Level: BCT-VII, BAI-V, BCS-V (A & B)

Instructors: Dr. Qazi Muhammad Zaigham Zia
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July 04, 2025¹

Time Allowed: 3 Hours

Total Marks: 50

Instructions:

Answer ALL questions. Show all the work. Answers without explanations will get partial credits only.

Question No. 01:

- (a) (2 marks) Verify that $y = \tan(x + c)$ is a one-parameter family of solutions of the differential equation

$$y' = 1 + y^2.$$

- (b) (3 marks) Determine whether the given differential equation is exact or not,

$$\cos x \, dx + \left(1 + \frac{2}{y}\right) \sin x \, dy = 0.$$

Question No. 02: (5 marks) Solve the given differential equation by using an appropriate substitution,

$$(y^2 + xy)dx - x^2 dy = 0.$$

Question No. 03: (5 marks) Solve the given differential equation by undetermined coefficients,

$$y'' - 10y' + 25y = 30x + 3.$$

Question No. 04: (5 marks) Solve the given differential equation by variation of parameters,

$$x^2 y'' + xy' - y = \frac{1}{1+x}.$$

Question No. 05: (5 marks) Find the charge on the capacitor in an LRC series circuit when $L = \frac{1}{4} \text{H}$, $R = 20\Omega$, $C = \frac{1}{300} \text{F}$, $E(t) = 0 \text{V}$, $q(0) = 4 \text{C}$, and $q'(0) = 0 \text{A}$. Is the charge on the capacitor ever equal to zero?

¹In case Ashura is observed on 4th July 2025, the exam will be held on 11th July 2025.

Question No. 06: (5 marks) Determine the singular points of the given differential equation. Classify each singular point as regular or irregular,

$$(x^2 + x - 6)y'' + (x + 3)y' + (x - 2)y = 0.$$

Question No. 07: (5 marks) Verify by direct substitution that $y = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} x^n$ is a solution of the following differential equation

$$(1 + x)y'' + y' = 0.$$

Question No. 08: (2.5 + 2.5 marks)

(a) Determine whether the given set of functions is linearly independent on the interval $(-\infty, \infty)$,

$$f_1(x) = x, f_2(x) = x^2, f_3(x) = 4x - 3x^2.$$

(b) Show that the given functions are orthogonal on the indicated interval,

$$f_1(x) = x, f_2(x) = x^2; [-2, 2].$$

Question No. 09: (03 + 07 marks)

(a) Determine the regions in the xy -plane for which the equation

$$\frac{\partial^2 u}{\partial x^2} + x \frac{\partial^2 u}{\partial y^2} = 0,$$

is hyperbolic, parabolic and elliptic.

(b) Solve

$$\begin{aligned} \frac{\partial u}{\partial t} &= k \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L, \quad t > 0 \\ u(0, t) &= 0, \quad u(L, t) = 0, \quad t > 0 \\ u(x, 0) &= 2x, \quad 0 < x < L. \end{aligned}$$

Good Luck
